

# The zeta function of $\mathbf{pm}$ counting all subgroups

## 1 Presentation

$\mathbf{pm}$  has presentation

$$\langle x, y, m \mid [x, y], m^2, x^m = x, y^m = y^{-1} \rangle.$$

## 2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathbf{pm}}(s) = (1 + 4 \cdot 2^{-s})\zeta(s)\zeta(s-1).$$

## 3 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{\mathbf{pm}}(s)$  is 2, with a simple pole at  $s = 2$ . Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to  $\mathbb{C}$ .