

The zeta function of \mathbf{pm} counting all subgroups

1 Presentation

\mathbf{pm} has presentation

$$\langle x, y, m \mid [x, y], m^2, x^m = x, y^m = y^{-1} \rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathbf{pm}}(s) = (1 + 4 \cdot 2^{-s})\zeta(s)\zeta(s-1).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathbf{pm}}(s)$ is 2, with a simple pole at $s = 2$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to \mathbb{C} .