

# The zeta function of $\mathbf{pg}$ counting normal subgroups

## 1 Presentation

$\mathbf{pg}$  has presentation

$$\langle x, y, t \mid [x, y], t^2 = x, y^t = y^{-1} \rangle.$$

## 2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathbf{pg}}^{\triangleleft}(s) = (1 + 2^{-s} - 4^{-s})\zeta(s) + (2^{-s} + 4^{-s})\zeta(s)^2.$$

## 3 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{\mathbf{pg}}^{\triangleleft}(s)$  is 1, with a double pole at  $s = 1$ . Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to  $\mathbb{C}$ .