

The zeta function of $\mathbf{p4mm}$ counting normal subgroups

1 Presentation

$\mathbf{p4mm}$ has presentation

$$\langle x, y, r, m \mid [x, y], r^4, m^2, y^r = x^{-1}, x^r = y, x^m = y, r^m = r^{-1} \rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathbf{p4mm}}^{\triangleleft}(s) = 1 + 7 \cdot 2^{-s} + 7 \cdot 4^{-s} + 6 \cdot 8^{-s} + 2 \cdot 16^{-s} + (8^{-s} + 16^{-s})\zeta(2s).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathbf{p4mm}}^{\triangleleft}(s)$ is $1/2$, with a simple pole at $s = 1/2$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to \mathbb{C} .