

The zeta function of $p4mm$ counting all subgroups

1 Presentation

$p4mm$ has presentation

$$\langle x, y, r, m \mid [x, y], r^4, m^2, y^r = x^{-1}, x^r = y, x^m = y, r^m = r^{-1} \rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\begin{aligned} \zeta_{p4mm}(s) &= (1 + 4 \cdot 2^{-s} + 4 \cdot 4^{-s})\zeta(2s - 2) \\ &\quad + (4 \cdot 4^{-s} + 5 \cdot 8^{-s} + 8 \cdot 16^{-s})\zeta(s)\zeta(s - 1) \\ &\quad + (2 \cdot 2^{-s} + 8 \cdot 4^{-s} + 4 \cdot 8^{-s})\zeta(s - 1)^2 + 4^{-s}\zeta(s - 1)\zeta(s - 2) \\ &\quad + 2^{-s}\zeta(s - 1)L(s - 1, \chi_4), \end{aligned}$$

where χ_4 is defined by

$$\chi_4(a) = \begin{cases} 1 & \text{if } a \equiv 1 \pmod{4} \\ -1 & \text{if } a \equiv -1 \pmod{4} \\ 0 & \text{otherwise} \end{cases},$$

and $L(s, \chi_4)$ is the Dirichlet L -function of χ_4 ,

$$L(s, \chi_4) = \sum_{n=1}^{\infty} \chi_4(n)n^{-s}.$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{p4mm}(s)$ is 3, with a simple pole at $s = 3$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to \mathbb{C} .