The zeta function of p4 counting normal subgroups

1 Presentation

p4 has presentation

$$\left\langle x, y, r \mid [x, y], r^4, y^r = x^{-1}, x^r = y \right\rangle .$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta^{\triangleleft}_{\mathbf{p4}}(s) = 1 + 3 \cdot 2^{-s} + 2 \cdot 4^{-s} + 2 \cdot 8^{-s} + 4^{-s}\zeta(s)L(s, \chi_4).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathbf{p4}}(s)$ is 1, with a simple pole at $s = 1$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.