

The zeta function of \mathfrak{p}_3 counting normal subgroups

1 Presentation

\mathfrak{p}_3 has presentation

$$\langle x, y, r \mid [x, y], r^3, x^r = x^{-1}y, y^r = x^{-1} \rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathfrak{p}_3}^{\triangleleft}(s) = 1 + 3 \cdot 3^{-s} + 3^{-s} \zeta(s) L(s, \chi_3).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathfrak{p}_3}^{\triangleleft}(s)$ is 1, with a simple pole at $s = 1$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to \mathbb{C} .