The zeta function of p2mm counting all subgroups

1 Presentation

p2mm has presentation

$$\left\langle x, y, p, q \mid [x, y], [p, q], p^2, q^2, x^p = x, x^q = x^{-1}, y^p = y^{-1}, y^q = y \right\rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\begin{aligned}\zeta_{\mathbf{p2mm}}(s) = {} & (1 + 8 \cdot 2^{-s} + 4 \cdot 4^{-s})\zeta(s-1)^2 + (2 \cdot 2^{-s} + 7 \cdot 4^{-s})\zeta(s)\zeta(s-1) \\ & + 2^{-s}\zeta(s-1)\zeta(s-2).\end{aligned}$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathbf{p2mm}}(s)$ is 3, with a simple pole at $s = 3$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.