

# The zeta function of $\mathfrak{p}2\mathfrak{mg}$ counting normal subgroups

## 1 Presentation

$\mathfrak{p}2\mathfrak{mg}$  has presentation

$$\langle x, y, m, t \mid [x, y], t^2, m^2 = y, x^t = x, x^m = x^{-1}, y^t = y^{-1}, m^t = m^{-1} \rangle.$$

## 2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathfrak{p}2\mathfrak{mg}}^{\triangleleft}(s) = 1 + 5 \cdot 2^{-s} + 2 \cdot 4^{-s} + (2 \cdot 2^{-s} + 2 \cdot 4^{-s} - 2 \cdot 8^{-s})\zeta(s) \\ + (4^{-s} + 8^{-s})\zeta(s)^2.$$

## 3 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{\mathfrak{p}2\mathfrak{mg}}^{\triangleleft}(s)$  is 1, with a double pole at  $s = 1$ . Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to  $\mathbb{C}$ .