# The zeta function of p2gg counting all subgroups

## 1 Presentation

**p2gg** has presentation

$$\left\langle x, y, u, v \mid [x, y], u^2 = x, v^2 = y, x^v = x^{-1}, y^u = y^{-1}, (uv)^2 \right\rangle .$$

## 2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\begin{aligned}\zeta_{\mathbf{p2gg}}(s) &= (1 - 2 \cdot 2^{-s})^2 \zeta(s-1)^2 + (2 \cdot 2^{-s} - 4^{-s})\zeta(s)\zeta(s-1) \\ &\quad + 2^{-s}\zeta(s-1)\zeta(s-2).\end{aligned}$$

## 3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathbf{p2gg}}(s)$ is 3, with a simple pole at $s = 3$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to $\mathbb{C}$.