

The zeta function of \mathfrak{p}_2 counting normal subgroups

1 Presentation

\mathfrak{p}_2 has presentation

$$\langle x, y, r \mid [x, y], r^2, x^r = x^{-1}, y^r = y^{-1} \rangle.$$

2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\zeta_{\mathfrak{p}_2}^{\triangleleft}(s) = 1 + 6 \cdot 2^{-s} + 4 \cdot 4^{-s} + 2^{-s} \zeta(s) \zeta(s-1).$$

3 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{\mathfrak{p}_2}^{\triangleleft}(s)$ is 2, with a simple pole at $s = 2$. Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to \mathbb{C} .