The zeta function of $\mathfrak{g}_{6,4}$ counting ideals

1 Presentation

$\mathfrak{g}_{6,4}$ has presentation

$$\langle x_1, x_2, x_3, x_4, x_5, x_6 \mid [x_1, x_2] = x_5, [x_1, x_3] = x_6, [x_2, x_4] = x_6 \rangle .$$

$\mathfrak{g}_{6,4}$ has nilpotency class 2.

2 The local zeta function

The local zeta function was first calculated by Grunewald, Segal & Smith. It is

$$\begin{aligned}\zeta^{\triangleleft}_{\mathfrak{g}_{6,4},p}(s) = {} & \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(3s-4)\zeta_p(5s-5)\zeta_p(6s-9) \\ & \times \zeta_p(8s-9)^{-1}.\end{aligned}$$

$\zeta^{\triangleleft}_{\mathfrak{g}_{6,4}}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta^{\triangleleft}_{\mathfrak{g}_{6,4},p}(s)\Big|_{p\to p^{-1}} = p^{15-10s}\zeta^{\triangleleft}_{\mathfrak{g}_{6,4},p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathfrak{g}_{6,4}}(s)$ is 4, with a simple pole at $s = 4$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta^{\triangleleft}_{\mathfrak{g}_{6,4}}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.

7 Notes

The Lie ring is sometimes written as $(F_{2,3}/\langle z \rangle) \cdot \mathbb{Z}$.