

# The zeta function of $\mathbf{c2mm}$ counting all subgroups

## 1 Presentation

$\mathbf{c2mm}$  has presentation

$$\langle x, y, m, r \mid [x, y], m^2, r^2, y^m = y^{-1}, x^m = xy, y^r = y^{-1}, x^r = x^{-1}, r^m = r^{-1} \rangle.$$

## 2 The zeta function itself

The zeta function was calculated by du Sautoy, McDermott and Smith. It is

$$\begin{aligned} \zeta_{\mathbf{c2mm}}(s) = & (1 + 8 \cdot 4^{-s})\zeta(s-1)^2 + (2 \cdot 2^{-s} - 4^{-s} + 8 \cdot 8^{-s})\zeta(s)\zeta(s-1) \\ & + 2^{-s}\zeta(s-1)\zeta(s-2). \end{aligned}$$

## 3 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{\mathbf{c2mm}}(s)$  is 3, with a simple pole at  $s = 3$ . Since this group is a finite extension of a free abelian group, its zeta function has meromorphic continuation to  $\mathbb{C}$ .