

The zeta function of $Q_5 \times \mathbb{Z}$ counting ideals

1 Presentation

$Q_5 \times \mathbb{Z}$ has presentation

$$\langle x_1, x_2, x_3, a, x_4, x_5 \mid [x_1, x_2] = x_4, [x_1, x_4] = x_5, [x_2, x_3] = x_5 \rangle.$$

$Q_5 \times \mathbb{Z}$ has nilpotency class 3.

2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\zeta_{Q_5 \times \mathbb{Z}, p}^{\triangleleft}(s) = \zeta_p(s) \zeta_p(s-1) \zeta_p(s-2) \zeta_p(s-3) \zeta_p(3s-4) \zeta_p(5s-5).$$

$\zeta_{Q_5 \times \mathbb{Z}}^{\triangleleft}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta_{Q_5 \times \mathbb{Z}, p}^{\triangleleft}(s) \Big|_{p \rightarrow p^{-1}} = p^{15-12s} \zeta_{Q_5 \times \mathbb{Z}, p}^{\triangleleft}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_{Q_5 \times \mathbb{Z}}^{\triangleleft}(s)$ is 4, with a simple pole at $s = 4$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta_{Q_5 \times \mathbb{Z}}^{\triangleleft}(s)$ has meromorphic continuation to the whole of \mathbb{C} .

7 Notes

This Lie ring comes from the only Lie algebra of dimension 5 not previously encountered by Luke Woodward. The subscript 5 comes from the dimension, but there isn't any significance in the letter Q .