# The zeta function of $M_3 \times \mathbb{Z}^2$ counting ideals

## 1 Presentation

$M_3 \times \mathbb{Z}^2$ has presentation

$$\langle z, x_1, x_2, a_1, a_2, x_3 \mid [z, x_1] = x_2, [z, x_2] = x_3 \rangle .$$

$M_3 \times \mathbb{Z}^2$ has nilpotency class 3.

## 2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\zeta^{\triangleleft}_{M_3 \times \mathbb{Z}^2, p}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(3s-4)\zeta_p(4s-4)\zeta_p(5s-5) \times \zeta_p(5s-4)^{-1}.$$

$\zeta^{\triangleleft}_{M_3 \times \mathbb{Z}^2}(s)$ is uniform.

## 3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta^{\triangleleft}_{M_3 \times \mathbb{Z}^2, p}(s)\Big|_{p \to p^{-1}} = p^{15-11s} \zeta^{\triangleleft}_{M_3 \times \mathbb{Z}^2, p}(s).$$

## 4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{M_3 \times \mathbb{Z}^2}(s)$ is 4, with a simple pole at $s = 4$.

## 5 Ghost zeta function

This zeta function is its own ghost.

## 6 Natural boundary

$\zeta^{\triangleleft}_{M_3 \times \mathbb{Z}^2}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.