

# The zeta function of $M_3$ counting ideals

## 1 Presentation

$M_3$  has presentation

$$\langle z, x_1, x_2, x_3 \mid [z, x_1] = x_2, [z, x_2] = x_3 \rangle.$$

$M_3$  has nilpotency class 3.

## 2 The local zeta function

The local zeta function was first calculated by Gareth Taylor. It is

$$\zeta_{M_3,p}^{\triangleleft}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(3s-2)\zeta_p(4s-2)\zeta_p(5s-3)\zeta_p(5s-2)^{-1}.$$

$\zeta_{M_3}^{\triangleleft}(s)$  is uniform.

## 3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta_{M_3,p}^{\triangleleft}(s) \Big|_{p \rightarrow p^{-1}} = p^{6-9s} \zeta_{M_3,p}^{\triangleleft}(s).$$

## 4 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{M_3}^{\triangleleft}(s)$  is 2, with a simple pole at  $s = 2$ .

## 5 Ghost zeta function

This zeta function is its own ghost.

## 6 Natural boundary

$\zeta_{M_3}^{\triangleleft}(s)$  has meromorphic continuation to the whole of  $\mathbb{C}$ .