

The zeta function of H counting ideals

1 Presentation

H has presentation

$$\langle x, y, z \mid [x, y] = z \rangle.$$

H has nilpotency class 2.

2 The local zeta function

The local zeta function was first calculated by Grunewald, Segal & Smith. It is

$$\zeta_{H,p}^{\triangleleft}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(3s-2).$$

$\zeta_H^{\triangleleft}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta_{H,p}^{\triangleleft}(s)|_{p \rightarrow p^{-1}} = -p^{3-5s}\zeta_{H,p}^{\triangleleft}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta_H^{\triangleleft}(s)$ is 2, with a simple pole at $s = 2$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta_H^{\triangleleft}(s)$ has meromorphic continuation to the whole of \mathbb{C} .