

# The zeta function of $H^2$ counting ideals

## 1 Presentation

$H^2$  has presentation

$$\langle x_1, x_2, y_1, y_2, z_1, z_2 \mid [x_1, y_1] = z_1, [x_2, y_2] = z_2 \rangle.$$

$H^2$  has nilpotency class 2.

## 2 The local zeta function

The local zeta function was first calculated by Grunewald, Segal & Smith. It is

$$\zeta_{H^2,p}^{\triangleleft}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(3s-4)^2\zeta_p(5s-5) \\ \times \zeta_p(5s-4)^{-1}.$$

$\zeta_{H^2}^{\triangleleft}(s)$  is uniform.

## 3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta_{H^2,p}^{\triangleleft}(s) \Big|_{p \rightarrow p^{-1}} = p^{15-10s} \zeta_{H^2,p}^{\triangleleft}(s).$$

## 4 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{H^2}^{\triangleleft}(s)$  is 4, with a simple pole at  $s = 4$ .

## 5 Ghost zeta function

This zeta function is its own ghost.

## 6 Natural boundary

$\zeta_{H^2}^{\triangleleft}(s)$  has meromorphic continuation to the whole of  $\mathbb{C}$ .