The zeta function of G_3 counting ideals

1 Presentation

G_3 has presentation

$$\langle z, x_1, x_2, y_1, y_2 \mid [z, x_1] = y_1, [z, x_2] = y_2 \rangle .$$

G_3 has nilpotency class 2.

2 The local zeta function

The local zeta function was first calculated by Grunewald, Segal & Smith. It is

$$\begin{aligned}\zeta^{\triangleleft}_{G_3,p}(s) = {} & \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(3s-3)\zeta_p(3s-4)\zeta_p(5s-6) \\ & \times \zeta_p(6s-6)^{-1}.\end{aligned}$$

$\zeta^{\triangleleft}_{G_3}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta^{\triangleleft}_{G_3,p}(s)\big|_{p \to p^{-1}} = -p^{10-8s}\zeta^{\triangleleft}_{G_3,p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{G_3}(s)$ is 3, with a simple pole at $s = 3$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta^{\triangleleft}_{G_3}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.