The zeta function of $\mathfrak{g}_{27B}$ counting ideals

1 Presentation

$\mathfrak{g}_{27B}$ has presentation

$$\left\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \;\middle|\; \begin{array}{c} [x_1, x_2] = x_6, [x_1, x_5] = x_7, \\ [x_2, x_3] = x_7, [x_3, x_4] = x_6 \end{array} \right\rangle.$$

$\mathfrak{g}_{27B}$ has nilpotency class 2.

2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\begin{aligned} \zeta^{\triangleleft}_{\mathfrak{g}_{27B},p}(s) = {} & \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(s-4)\zeta_p(5s-5)\zeta_p(5s-6) \\ & \times \zeta_p(7s-10)\zeta_p(10s-10)^{-1}. \end{aligned}$$

$\zeta^{\triangleleft}_{\mathfrak{g}_{27B}}(s)$ is uniform.

3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta^{\triangleleft}_{\mathfrak{g}_{27B},p}(s)\big|_{p\to p^{-1}} = -p^{21-12s}\zeta^{\triangleleft}_{\mathfrak{g}_{27B},p}(s).$$

4 Abscissa of convergence and order of pole

The abscissa of convergence of $\zeta^{\triangleleft}_{\mathfrak{g}_{27B}}(s)$ is 5, with a simple pole at $s = 5$.

5 Ghost zeta function

This zeta function is its own ghost.

6 Natural boundary

$\zeta^{\triangleleft}_{\mathfrak{g}_{27B}}(s)$ has meromorphic continuation to the whole of $\mathbb{C}$.