

# The zeta function of $\mathfrak{g}_{247B}$ counting ideals

## 1 Presentation

$\mathfrak{g}_{247B}$  has presentation

$$\left\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \left| \begin{array}{l} [x_1, x_2] = x_4, [x_1, x_3] = x_5, \\ [x_1, x_4] = x_6, [x_3, x_5] = x_7 \end{array} \right. \right\rangle.$$

$\mathfrak{g}_{247B}$  has nilpotency class 3.

## 2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\begin{aligned} \zeta_{\mathfrak{g}_{247B}, p}^{\triangleleft}(s) &= \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(3s-4)\zeta_p(4s-3)\zeta_p(5s-5)\zeta_p(5s-6) \\ &\quad \times \zeta_p(6s-5)\zeta_p(6s-6)\zeta_p(7s-6)\zeta_p(7s-7)\zeta_p(8s-7) \\ &\quad \times \zeta_p(8s-8)\zeta_p(9s-10)\zeta_p(9s-11)\zeta_p(10s-9)\zeta_p(10s-11) \\ &\quad \times \zeta_p(11s-10)\zeta_p(11s-12)\zeta_p(12s-12)\zeta_p(13s-13) \\ &\quad \times W(p, p^{-s}) \end{aligned}$$

where  $W(X, Y)$  is

$$\begin{aligned} &1 + X^3Y^3 + X^4Y^4 - X^3Y^5 - X^4Y^5 + X^5Y^5 - X^4Y^6 + X^6Y^6 + X^4Y^7 \\ &- 2X^5Y^7 - 2X^6Y^7 + X^8Y^7 - 2X^6Y^8 - 2X^7Y^8 + X^8Y^8 + X^6Y^9 - 5X^8Y^9 \\ &- X^{10}Y^9 + X^7Y^{10} - 3X^8Y^{10} - 2X^9Y^{10} - X^{11}Y^{10} - X^7Y^{11} + 5X^8Y^{11} \\ &- 2X^9Y^{11} - 2X^{10}Y^{11} - 7X^{11}Y^{11} + 2X^{12}Y^{11} + 4X^9Y^{12} - 5X^{10}Y^{12} \\ &+ X^{11}Y^{12} - 3X^{12}Y^{12} - X^{14}Y^{12} - X^9Y^{13} + 6X^{10}Y^{13} + 6X^{11}Y^{13} \\ &- 9X^{12}Y^{13} - X^{13}Y^{13} - 4X^{14}Y^{13} + X^{10}Y^{14} + 2X^{11}Y^{14} + 6X^{12}Y^{14} \\ &- 6X^{13}Y^{14} + X^{14}Y^{14} - 2X^{15}Y^{14} - X^{16}Y^{14} - X^{10}Y^{15} - X^{11}Y^{15} \\ &+ 7X^{12}Y^{15} + 5X^{13}Y^{15} + 10X^{14}Y^{15} - 11X^{15}Y^{15} - X^{16}Y^{15} - X^{17}Y^{15} \\ &- 2X^{12}Y^{16} + 10X^{13}Y^{16} - 2X^{14}Y^{16} + 9X^{15}Y^{16} - 3X^{16}Y^{16} + X^{17}Y^{16} \\ &- 2X^{18}Y^{16} - 3X^{13}Y^{17} + X^{14}Y^{17} + 22X^{15}Y^{17} + 2X^{16}Y^{17} - X^{18}Y^{17} \\ &- 2X^{19}Y^{17} - 2X^{13}Y^{18} + X^{14}Y^{18} - 4X^{15}Y^{18} + 19X^{16}Y^{18} + 4X^{17}Y^{18} \\ &+ 7X^{18}Y^{18} - X^{19}Y^{18} + X^{13}Y^{19} - 2X^{14}Y^{19} - 10X^{15}Y^{19} + 4X^{16}Y^{19} \\ &+ 14X^{17}Y^{19} + 16X^{18}Y^{19} + 6X^{19}Y^{19} - 2X^{20}Y^{19} + X^{21}Y^{19} + 2X^{15}Y^{20} \\ &- 12X^{16}Y^{20} + 2X^{17}Y^{20} + 4X^{18}Y^{20} + 16X^{19}Y^{20} + 7X^{20}Y^{20} + 7X^{21}Y^{20} \end{aligned}$$

$$\begin{aligned}
& -X^{16}Y^{21} - 8X^{17}Y^{21} - 16X^{18}Y^{21} + 16X^{19}Y^{21} + 20X^{20}Y^{21} + 7X^{21}Y^{21} \\
& + 5X^{22}Y^{21} + X^{16}Y^{22} - 4X^{17}Y^{22} - 23X^{19}Y^{22} + 2X^{20}Y^{22} + 8X^{21}Y^{22} \\
& + 17X^{22}Y^{22} + 7X^{23}Y^{22} + 2X^{24}Y^{22} + X^{25}Y^{22} + 2X^{17}Y^{23} - 16X^{19}Y^{23} \\
& - 19X^{20}Y^{23} - 3X^{21}Y^{23} + 6X^{22}Y^{23} + 20X^{23}Y^{23} + 5X^{24}Y^{23} + 3X^{25}Y^{23} \\
& + X^{26}Y^{23} - X^{18}Y^{24} + 8X^{19}Y^{24} - 15X^{20}Y^{24} - 24X^{21}Y^{24} - 15X^{22}Y^{24} \\
& - 4X^{23}Y^{24} + 14X^{24}Y^{24} + 5X^{25}Y^{24} + 7X^{26}Y^{24} + X^{27}Y^{24} + X^{19}Y^{25} \\
& + 5X^{20}Y^{25} - 7X^{21}Y^{25} - 37X^{22}Y^{25} - 21X^{23}Y^{25} + X^{24}Y^{25} + 4X^{25}Y^{25} \\
& + 7X^{26}Y^{25} + 6X^{27}Y^{25} + 2X^{20}Y^{26} + 5X^{21}Y^{26} + 5X^{22}Y^{26} - 33X^{23}Y^{26} \\
& - 44X^{24}Y^{26} - 11X^{25}Y^{26} - 5X^{26}Y^{26} + 10X^{27}Y^{26} + 4X^{28}Y^{26} + 2X^{29}Y^{26} \\
& + 2X^{21}Y^{27} + 11X^{22}Y^{27} - 2X^{23}Y^{27} - 33X^{24}Y^{27} - 32X^{25}Y^{27} - 28X^{26}Y^{27} \\
& - 9X^{27}Y^{27} + 8X^{28}Y^{27} + 2X^{30}Y^{27} - X^{22}Y^{28} + 13X^{23}Y^{28} + 20X^{24}Y^{28} \\
& - 28X^{25}Y^{28} - 38X^{26}Y^{28} - 50X^{27}Y^{28} - 11X^{28}Y^{28} - 4X^{29}Y^{28} + 4X^{30}Y^{28} \\
& + X^{31}Y^{28} - X^{22}Y^{29} + 5X^{23}Y^{29} + 12X^{24}Y^{29} + 16X^{25}Y^{29} - 10X^{26}Y^{29} \\
& - 39X^{27}Y^{29} - 47X^{28}Y^{29} - 12X^{29}Y^{29} - 17X^{30}Y^{29} + 6X^{31}Y^{29} - 2X^{32}Y^{29} \\
& - 2X^{23}Y^{30} + X^{24}Y^{30} + 17X^{25}Y^{30} + 27X^{26}Y^{30} + 18X^{27}Y^{30} - 44X^{28}Y^{30} \\
& - 47X^{29}Y^{30} - 37X^{30}Y^{30} - 11X^{31}Y^{30} - 4X^{32}Y^{30} + X^{33}Y^{30} - X^{24}Y^{31} \\
& + 20X^{26}Y^{31} + 36X^{27}Y^{31} + 13X^{28}Y^{31} - 27X^{29}Y^{31} - 20X^{30}Y^{31} - 57X^{31}Y^{31} \\
& - 4X^{32}Y^{31} - 10X^{33}Y^{31} - 4X^{26}Y^{32} + 6X^{27}Y^{32} + 66X^{28}Y^{32} + 34X^{29}Y^{32} \\
& - 43X^{31}Y^{32} - 32X^{32}Y^{32} - 27X^{33}Y^{32} - 10X^{34}Y^{32} - 2X^{26}Y^{33} - 3X^{27}Y^{33} \\
& + 5X^{28}Y^{33} + 56X^{29}Y^{33} + 41X^{30}Y^{33} + 47X^{31}Y^{33} - 48X^{32}Y^{33} - 10X^{33}Y^{33} \\
& - 34X^{34}Y^{33} - 10X^{35}Y^{33} - 2X^{36}Y^{33} - X^{37}Y^{33} - 11X^{28}Y^{34} + 3X^{29}Y^{34} \\
& + 48X^{30}Y^{34} + 86X^{31}Y^{34} + 45X^{32}Y^{34} - 4X^{33}Y^{34} - 33X^{34}Y^{34} - 22X^{35}Y^{34} \\
& - 11X^{36}Y^{34} - 5X^{37}Y^{34} - X^{28}Y^{35} - 13X^{29}Y^{35} + 5X^{30}Y^{35} + 26X^{31}Y^{35} \\
& + 99X^{32}Y^{35} + 31X^{33}Y^{35} + 63X^{34}Y^{35} - 28X^{35}Y^{35} - 17X^{36}Y^{35} - 10X^{37}Y^{35} \\
& - 6X^{38}Y^{35} - X^{29}Y^{36} - 17X^{30}Y^{36} - 18X^{31}Y^{36} + 29X^{32}Y^{36} + 95X^{33}Y^{36} \\
& + 94X^{34}Y^{36} + 45X^{35}Y^{36} - 3X^{36}Y^{36} - 10X^{37}Y^{36} - 17X^{38}Y^{36} - 4X^{39}Y^{36} \\
& - 2X^{30}Y^{37} - 17X^{31}Y^{37} - 29X^{32}Y^{37} + 31X^{33}Y^{37} + 39X^{34}Y^{37} + 131X^{35}Y^{37} \\
& + 51X^{36}Y^{37} + 34X^{37}Y^{37} + 3X^{38}Y^{37} - 15X^{39}Y^{37} - 3X^{40}Y^{37} - X^{41}Y^{37} \\
& + X^{31}Y^{38} - 18X^{32}Y^{38} - 43X^{33}Y^{38} - 9X^{34}Y^{38} + 59X^{35}Y^{38} + 118X^{36}Y^{38} \\
& + 85X^{37}Y^{38} + 43X^{38}Y^{38} + 4X^{39}Y^{38} - 7X^{40}Y^{38} - 2X^{41}Y^{38} - X^{42}Y^{38} \\
& + X^{32}Y^{39} - 19X^{33}Y^{39} - 40X^{34}Y^{39} - 60X^{35}Y^{39} + 70X^{36}Y^{39} + 55X^{37}Y^{39} \\
& + 109X^{38}Y^{39} + 66X^{39}Y^{39} + 20X^{40}Y^{39} + 3X^{41}Y^{39} - 5X^{42}Y^{39} + X^{43}Y^{39} \\
& + 2X^{33}Y^{40} - 12X^{34}Y^{40} - 51X^{35}Y^{40} - 89X^{36}Y^{40} + 22X^{37}Y^{40} + 69X^{38}Y^{40} \\
& + 114X^{39}Y^{40} + 65X^{40}Y^{40} + 31X^{41}Y^{40} + 10X^{42}Y^{40} - 6X^{43}Y^{40} + X^{44}Y^{40}
\end{aligned}$$

$$\begin{aligned}
& + X^{33}Y^{41} + 4X^{34}Y^{41} - 8X^{35}Y^{41} - 66X^{36}Y^{41} - 73X^{37}Y^{41} - 59X^{38}Y^{41} \\
& + 40X^{39}Y^{41} + 99X^{40}Y^{41} + 66X^{41}Y^{41} + 58X^{42}Y^{41} + 9X^{43}Y^{41} + 4X^{44}Y^{41} \\
& - 2X^{45}Y^{41} + 6X^{35}Y^{42} - 3X^{36}Y^{42} - 62X^{37}Y^{42} - 93X^{38}Y^{42} - 96X^{39}Y^{42} \\
& + 3X^{40}Y^{42} + 84X^{41}Y^{42} + 67X^{42}Y^{42} + 58X^{43}Y^{42} + 5X^{44}Y^{42} + 10X^{45}Y^{42} \\
& - X^{46}Y^{42} + X^{35}Y^{43} + 12X^{36}Y^{43} - X^{37}Y^{43} - 41X^{38}Y^{43} - 123X^{39}Y^{43} \\
& - 111X^{40}Y^{43} - 47X^{41}Y^{43} + 6X^{42}Y^{43} + 82X^{43}Y^{43} + 54X^{44}Y^{43} + 26X^{45}Y^{43} \\
& + 4X^{46}Y^{43} + X^{47}Y^{43} + X^{36}Y^{44} + 11X^{37}Y^{44} + 8X^{38}Y^{44} - 19X^{39}Y^{44} \\
& - 123X^{40}Y^{44} - 162X^{41}Y^{44} - 78X^{42}Y^{44} - 5X^{43}Y^{44} + 59X^{44}Y^{44} \\
& + 33X^{45}Y^{44} + 35X^{46}Y^{44} + 2X^{48}Y^{44} + X^{37}Y^{45} + 12X^{38}Y^{45} + 31X^{39}Y^{45} \\
& - 20X^{40}Y^{45} - 105X^{41}Y^{45} - 141X^{42}Y^{45} - 171X^{43}Y^{45} - 25X^{44}Y^{45} \\
& - 8X^{45}Y^{45} + 47X^{46}Y^{45} + 33X^{47}Y^{45} + 7X^{48}Y^{45} + X^{49}Y^{45} + 2X^{38}Y^{46} \\
& + 5X^{39}Y^{46} + 40X^{40}Y^{46} + 25X^{41}Y^{46} - 94X^{42}Y^{46} - 146X^{43}Y^{46} \\
& - 200X^{44}Y^{46} - 58X^{45}Y^{46} - 35X^{46}Y^{46} + 45X^{47}Y^{46} + 9X^{48}Y^{46} + 13X^{49}Y^{46} \\
& + X^{50}Y^{46} + 14X^{40}Y^{47} + 44X^{41}Y^{47} + 47X^{42}Y^{47} - 41X^{43}Y^{47} - 177X^{44}Y^{47} \\
& - 139X^{45}Y^{47} - 161X^{46}Y^{47} - 62X^{47}Y^{47} + 15X^{48}Y^{47} + 19X^{49}Y^{47} \\
& + 12X^{50}Y^{47} + X^{51}Y^{47} + X^{39}Y^{48} - X^{40}Y^{48} + 10X^{41}Y^{48} + 43X^{42}Y^{48} \\
& + 60X^{43}Y^{48} + 42X^{44}Y^{48} - 154X^{45}Y^{48} - 159X^{46}Y^{48} - 187X^{47}Y^{48} \\
& - 41X^{48}Y^{48} - 40X^{49}Y^{48} + 21X^{50}Y^{48} + 4X^{51}Y^{48} + 2X^{52}Y^{48} - 2X^{41}Y^{49} \\
& + 8X^{42}Y^{49} + 60X^{43}Y^{49} + 91X^{44}Y^{49} + 23X^{45}Y^{49} - 20X^{46}Y^{49} - 182X^{47}Y^{49} \\
& - 181X^{48}Y^{49} - 96X^{49}Y^{49} - 55X^{50}Y^{49} + 12X^{51}Y^{49} + 7X^{52}Y^{49} + X^{53}Y^{49} \\
& - X^{42}Y^{50} + 2X^{43}Y^{50} + 37X^{44}Y^{50} + 125X^{45}Y^{50} + 72X^{46}Y^{50} + 47X^{47}Y^{50} \\
& - 174X^{48}Y^{50} - 125X^{49}Y^{50} - 152X^{50}Y^{50} - 35X^{51}Y^{50} - 15X^{52}Y^{50} \\
& + X^{53}Y^{50} + 2X^{54}Y^{50} - 6X^{43}Y^{51} - 4X^{44}Y^{51} + 53X^{45}Y^{51} + 103X^{46}Y^{51} \\
& + 165X^{47}Y^{51} + 46X^{48}Y^{51} - 74X^{49}Y^{51} - 103X^{50}Y^{51} - 171X^{51}Y^{51} \\
& - 72X^{52}Y^{51} - 13X^{53}Y^{51} + 2X^{54}Y^{51} + X^{55}Y^{51} - X^{44}Y^{52} - 15X^{45}Y^{52} \\
& + 34X^{46}Y^{52} + 82X^{47}Y^{52} + 224X^{48}Y^{52} + 95X^{49}Y^{52} + 24X^{50}Y^{52} \\
& - 131X^{51}Y^{52} - 87X^{52}Y^{52} - 93X^{53}Y^{52} - 27X^{54}Y^{52} - 2X^{55}Y^{52} - X^{56}Y^{52} \\
& - X^{44}Y^{53} - 6X^{45}Y^{53} - 14X^{46}Y^{53} + 4X^{47}Y^{53} + 122X^{48}Y^{53} + 181X^{49}Y^{53} \\
& + 165X^{50}Y^{53} + 114X^{51}Y^{53} - 74X^{52}Y^{53} - 86X^{53}Y^{53} - 102X^{54}Y^{53} \\
& - 32X^{55}Y^{53} - 2X^{56}Y^{53} - X^{57}Y^{53} - X^{45}Y^{54} - 5X^{46}Y^{54} - 12X^{47}Y^{54} \\
& - 35X^{48}Y^{54} + 91X^{49}Y^{54} + 173X^{50}Y^{54} + 260X^{51}Y^{54} + 114X^{52}Y^{54} \\
& + 29X^{53}Y^{54} - 70X^{54}Y^{54} - 77X^{55}Y^{54} - 30X^{56}Y^{54} - 15X^{57}Y^{54} + X^{58}Y^{54} \\
& - X^{46}Y^{55} - 6X^{47}Y^{55} - 32X^{48}Y^{55} - 14X^{49}Y^{55} + 32X^{50}Y^{55} + 171X^{51}Y^{55} \\
& + 262X^{52}Y^{55} + 166X^{53}Y^{55} + 114X^{54}Y^{55} - 34X^{55}Y^{55} - 75X^{56}Y^{55}
\end{aligned}$$

$$\begin{aligned}
& -34X^{57}Y^{55} - 13X^{58}Y^{55} - X^{47}Y^{56} - 39X^{49}Y^{56} - 26X^{50}Y^{56} - 43X^{51}Y^{56} \\
& + 142X^{52}Y^{56} + 243X^{53}Y^{56} + 257X^{54}Y^{56} + 128X^{55}Y^{56} + 14X^{56}Y^{56} \\
& - 21X^{57}Y^{56} - 40X^{58}Y^{56} - 13X^{59}Y^{56} - 3X^{60}Y^{56} - 4X^{49}Y^{57} - 37X^{50}Y^{57} \\
& - 62X^{51}Y^{57} - 39X^{52}Y^{57} + 95X^{53}Y^{57} + 176X^{54}Y^{57} + 284X^{55}Y^{57} \\
& + 180X^{56}Y^{57} + 70X^{57}Y^{57} - 13X^{58}Y^{57} - 37X^{59}Y^{57} - 7X^{60}Y^{57} - 3X^{61}Y^{57} \\
& - 3X^{50}Y^{58} - 17X^{51}Y^{58} - 94X^{52}Y^{58} - 87X^{53}Y^{58} + 3X^{54}Y^{58} + 161X^{55}Y^{58} \\
& + 263X^{56}Y^{58} + 193X^{57}Y^{58} + 143X^{58}Y^{58} - 3X^{59}Y^{58} - 11X^{60}Y^{58} \\
& - 13X^{61}Y^{58} - 3X^{62}Y^{58} + X^{50}Y^{59} - 3X^{51}Y^{59} - 22X^{52}Y^{59} - 86X^{53}Y^{59} \\
& - 121X^{54}Y^{59} - 62X^{55}Y^{59} + 116X^{56}Y^{59} + 199X^{57}Y^{59} + 200X^{58}Y^{59} \\
& + 195X^{59}Y^{59} + 6X^{60}Y^{59} - X^{61}Y^{59} - 12X^{62}Y^{59} - X^{63}Y^{59} + 5X^{52}Y^{60} \\
& - 18X^{53}Y^{60} - 68X^{54}Y^{60} - 162X^{55}Y^{60} - 145X^{56}Y^{60} + 27X^{57}Y^{60} \\
& + 142X^{58}Y^{60} + 222X^{59}Y^{60} + 160X^{60}Y^{60} + 74X^{61}Y^{60} + 5X^{62}Y^{60} \\
& - 7X^{63}Y^{60} - 2X^{64}Y^{60} + X^{52}Y^{61} + 5X^{53}Y^{61} - 10X^{54}Y^{61} - 66X^{55}Y^{61} \\
& - 179X^{56}Y^{61} - 152X^{57}Y^{61} - 78X^{58}Y^{61} + 47X^{59}Y^{61} + 226X^{60}Y^{61} \\
& + 130X^{61}Y^{61} + 106X^{62}Y^{61} + 5X^{63}Y^{61} - X^{64}Y^{61} - 2X^{65}Y^{61} + X^{53}Y^{62} \\
& + 7X^{54}Y^{62} + 4X^{55}Y^{62} - 39X^{56}Y^{62} - 185X^{57}Y^{62} - 206X^{58}Y^{62} \\
& - 180X^{59}Y^{62} + 7X^{60}Y^{62} + 117X^{61}Y^{62} + 161X^{62}Y^{62} + 93X^{63}Y^{62} \\
& + 30X^{64}Y^{62} - X^{66}Y^{62} + X^{54}Y^{63} + 14X^{55}Y^{63} + 9X^{56}Y^{63} - 38X^{57}Y^{63} \\
& - 127X^{58}Y^{63} - 235X^{59}Y^{63} - 246X^{60}Y^{63} - 34X^{61}Y^{63} - 13X^{62}Y^{63} \\
& + 176X^{63}Y^{63} + 72X^{64}Y^{63} + 44X^{65}Y^{63} - X^{66}Y^{63} + X^{67}Y^{63} + 13X^{56}Y^{64} \\
& + 34X^{57}Y^{64} - 23X^{58}Y^{64} - 77X^{59}Y^{64} - 263X^{60}Y^{64} - 270X^{61}Y^{64} \\
& - 173X^{62}Y^{64} - 23X^{63}Y^{64} + 99X^{64}Y^{64} + 81X^{65}Y^{64} + 49X^{66}Y^{64} + 3X^{67}Y^{64} \\
& + X^{68}Y^{64} + 2X^{56}Y^{65} + 9X^{57}Y^{65} + 33X^{58}Y^{65} + 25X^{59}Y^{65} - 68X^{60}Y^{65} \\
& - 230X^{61}Y^{65} - 214X^{62}Y^{65} - 294X^{63}Y^{65} - 54X^{64}Y^{65} + 15X^{65}Y^{65} \\
& + 88X^{66}Y^{65} + 31X^{67}Y^{65} + 11X^{68}Y^{65} + X^{69}Y^{65} + 2X^{57}Y^{66} + 13X^{58}Y^{66} \\
& + 24X^{59}Y^{66} + 84X^{60}Y^{66} - 33X^{61}Y^{66} - 159X^{62}Y^{66} - 275X^{63}Y^{66} \\
& - 275X^{64}Y^{66} - 159X^{65}Y^{66} - 33X^{66}Y^{66} + 84X^{67}Y^{66} + 24X^{68}Y^{66} \\
& + 13X^{69}Y^{66} + 2X^{70}Y^{66} + X^{58}Y^{67} + 11X^{59}Y^{67} + 31X^{60}Y^{67} + 88X^{61}Y^{67} \\
& + 15X^{62}Y^{67} - 54X^{63}Y^{67} - 294X^{64}Y^{67} - 214X^{65}Y^{67} - 230X^{66}Y^{67} \\
& - 68X^{67}Y^{67} + 25X^{68}Y^{67} + 33X^{69}Y^{67} + 9X^{70}Y^{67} + 2X^{71}Y^{67} + X^{59}Y^{68} \\
& + 3X^{60}Y^{68} + 49X^{61}Y^{68} + 81X^{62}Y^{68} + 99X^{63}Y^{68} - 23X^{64}Y^{68} - 173X^{65}Y^{68} \\
& - 270X^{66}Y^{68} - 263X^{67}Y^{68} - 77X^{68}Y^{68} - 23X^{69}Y^{68} + 34X^{70}Y^{68} \\
& + 13X^{71}Y^{68} + X^{60}Y^{69} - X^{61}Y^{69} + 44X^{62}Y^{69} + 72X^{63}Y^{69} + 176X^{64}Y^{69} \\
& - 13X^{65}Y^{69} - 34X^{66}Y^{69} - 246X^{67}Y^{69} - 235X^{68}Y^{69} - 127X^{69}Y^{69}
\end{aligned}$$

$$\begin{aligned}
& -38X^{70}Y^{69} + 9X^{71}Y^{69} + 14X^{72}Y^{69} + X^{73}Y^{69} - X^{61}Y^{70} + 30X^{63}Y^{70} \\
& + 93X^{64}Y^{70} + 161X^{65}Y^{70} + 117X^{66}Y^{70} + 7X^{67}Y^{70} - 180X^{68}Y^{70} \\
& - 206X^{69}Y^{70} - 185X^{70}Y^{70} - 39X^{71}Y^{70} + 4X^{72}Y^{70} + 7X^{73}Y^{70} + X^{74}Y^{70} \\
& - 2X^{62}Y^{71} - X^{63}Y^{71} + 5X^{64}Y^{71} + 106X^{65}Y^{71} + 130X^{66}Y^{71} + 226X^{67}Y^{71} \\
& + 47X^{68}Y^{71} - 78X^{69}Y^{71} - 152X^{70}Y^{71} - 179X^{71}Y^{71} - 66X^{72}Y^{71} \\
& - 10X^{73}Y^{71} + 5X^{74}Y^{71} + X^{75}Y^{71} - 2X^{63}Y^{72} - 7X^{64}Y^{72} + 5X^{65}Y^{72} \\
& + 74X^{66}Y^{72} + 160X^{67}Y^{72} + 222X^{68}Y^{72} + 142X^{69}Y^{72} + 27X^{70}Y^{72} \\
& - 145X^{71}Y^{72} - 162X^{72}Y^{72} - 68X^{73}Y^{72} - 18X^{74}Y^{72} + 5X^{75}Y^{72} - X^{64}Y^{73} \\
& - 12X^{65}Y^{73} - X^{66}Y^{73} + 6X^{67}Y^{73} + 195X^{68}Y^{73} + 200X^{69}Y^{73} + 199X^{70}Y^{73} \\
& + 116X^{71}Y^{73} - 62X^{72}Y^{73} - 121X^{73}Y^{73} - 86X^{74}Y^{73} - 22X^{75}Y^{73} \\
& - 3X^{76}Y^{73} + X^{77}Y^{73} - 3X^{65}Y^{74} - 13X^{66}Y^{74} - 11X^{67}Y^{74} - 3X^{68}Y^{74} \\
& + 143X^{69}Y^{74} + 193X^{70}Y^{74} + 263X^{71}Y^{74} + 161X^{72}Y^{74} + 3X^{73}Y^{74} \\
& - 87X^{74}Y^{74} - 94X^{75}Y^{74} - 17X^{76}Y^{74} - 3X^{77}Y^{74} - 3X^{66}Y^{75} - 7X^{67}Y^{75} \\
& - 37X^{68}Y^{75} - 13X^{69}Y^{75} + 70X^{70}Y^{75} + 180X^{71}Y^{75} + 284X^{72}Y^{75} \\
& + 176X^{73}Y^{75} + 95X^{74}Y^{75} - 39X^{75}Y^{75} - 62X^{76}Y^{75} - 37X^{77}Y^{75} - 4X^{78}Y^{75} \\
& - 3X^{67}Y^{76} - 13X^{68}Y^{76} - 40X^{69}Y^{76} - 21X^{70}Y^{76} + 14X^{71}Y^{76} + 128X^{72}Y^{76} \\
& + 257X^{73}Y^{76} + 243X^{74}Y^{76} + 142X^{75}Y^{76} - 43X^{76}Y^{76} - 26X^{77}Y^{76} \\
& - 39X^{78}Y^{76} - X^{80}Y^{76} - 13X^{69}Y^{77} - 34X^{70}Y^{77} - 75X^{71}Y^{77} - 34X^{72}Y^{77} \\
& + 114X^{73}Y^{77} + 166X^{74}Y^{77} + 262X^{75}Y^{77} + 171X^{76}Y^{77} + 32X^{77}Y^{77} \\
& - 14X^{78}Y^{77} - 32X^{79}Y^{77} - 6X^{80}Y^{77} - X^{81}Y^{77} + X^{69}Y^{78} - 15X^{70}Y^{78} \\
& - 30X^{71}Y^{78} - 77X^{72}Y^{78} - 70X^{73}Y^{78} + 29X^{74}Y^{78} + 114X^{75}Y^{78} \\
& + 260X^{76}Y^{78} + 173X^{77}Y^{78} + 91X^{78}Y^{78} - 35X^{79}Y^{78} - 12X^{80}Y^{78} \\
& - 5X^{81}Y^{78} - X^{82}Y^{78} - X^{70}Y^{79} - 2X^{71}Y^{79} - 32X^{72}Y^{79} - 102X^{73}Y^{79} \\
& - 86X^{74}Y^{79} - 74X^{75}Y^{79} + 114X^{76}Y^{79} + 165X^{77}Y^{79} + 181X^{78}Y^{79} \\
& + 122X^{79}Y^{79} + 4X^{80}Y^{79} - 14X^{81}Y^{79} - 6X^{82}Y^{79} - X^{83}Y^{79} - X^{71}Y^{80} \\
& - 2X^{72}Y^{80} - 27X^{73}Y^{80} - 93X^{74}Y^{80} - 87X^{75}Y^{80} - 131X^{76}Y^{80} + 24X^{77}Y^{80} \\
& + 95X^{78}Y^{80} + 224X^{79}Y^{80} + 82X^{80}Y^{80} + 34X^{81}Y^{80} - 15X^{82}Y^{80} - X^{83}Y^{80} \\
& + X^{72}Y^{81} + 2X^{73}Y^{81} - 13X^{74}Y^{81} - 72X^{75}Y^{81} - 171X^{76}Y^{81} - 103X^{77}Y^{81} \\
& - 74X^{78}Y^{81} + 46X^{79}Y^{81} + 165X^{80}Y^{81} + 103X^{81}Y^{81} + 53X^{82}Y^{81} \\
& - 4X^{83}Y^{81} - 6X^{84}Y^{81} + 2X^{73}Y^{82} + X^{74}Y^{82} - 15X^{75}Y^{82} - 35X^{76}Y^{82} \\
& - 152X^{77}Y^{82} - 125X^{78}Y^{82} - 174X^{79}Y^{82} + 47X^{80}Y^{82} + 72X^{81}Y^{82} \\
& + 125X^{82}Y^{82} + 37X^{83}Y^{82} + 2X^{84}Y^{82} - X^{85}Y^{82} + X^{74}Y^{83} + 7X^{75}Y^{83} \\
& + 12X^{76}Y^{83} - 55X^{77}Y^{83} - 96X^{78}Y^{83} - 181X^{79}Y^{83} - 182X^{80}Y^{83} \\
& - 20X^{81}Y^{83} + 23X^{82}Y^{83} + 91X^{83}Y^{83} + 60X^{84}Y^{83} + 8X^{85}Y^{83} - 2X^{86}Y^{83}
\end{aligned}$$

$$\begin{aligned}
& + 2X^{75}Y^{84} + 4X^{76}Y^{84} + 21X^{77}Y^{84} - 40X^{78}Y^{84} - 41X^{79}Y^{84} - 187X^{80}Y^{84} \\
& - 159X^{81}Y^{84} - 154X^{82}Y^{84} + 42X^{83}Y^{84} + 60X^{84}Y^{84} + 43X^{85}Y^{84} \\
& + 10X^{86}Y^{84} - X^{87}Y^{84} + X^{88}Y^{84} + X^{76}Y^{85} + 12X^{77}Y^{85} + 19X^{78}Y^{85} \\
& + 15X^{79}Y^{85} - 62X^{80}Y^{85} - 161X^{81}Y^{85} - 139X^{82}Y^{85} - 177X^{83}Y^{85} \\
& - 41X^{84}Y^{85} + 47X^{85}Y^{85} + 44X^{86}Y^{85} + 14X^{87}Y^{85} + X^{77}Y^{86} + 13X^{78}Y^{86} \\
& + 9X^{79}Y^{86} + 45X^{80}Y^{86} - 35X^{81}Y^{86} - 58X^{82}Y^{86} - 200X^{83}Y^{86} \\
& - 146X^{84}Y^{86} - 94X^{85}Y^{86} + 25X^{86}Y^{86} + 40X^{87}Y^{86} + 5X^{88}Y^{86} + 2X^{89}Y^{86} \\
& + X^{78}Y^{87} + 7X^{79}Y^{87} + 33X^{80}Y^{87} + 47X^{81}Y^{87} - 8X^{82}Y^{87} - 25X^{83}Y^{87} \\
& - 171X^{84}Y^{87} - 141X^{85}Y^{87} - 105X^{86}Y^{87} - 20X^{87}Y^{87} + 31X^{88}Y^{87} \\
& + 12X^{89}Y^{87} + X^{90}Y^{87} + 2X^{79}Y^{88} + 35X^{81}Y^{88} + 33X^{82}Y^{88} + 59X^{83}Y^{88} \\
& - 5X^{84}Y^{88} - 78X^{85}Y^{88} - 162X^{86}Y^{88} - 123X^{87}Y^{88} - 19X^{88}Y^{88} + 8X^{89}Y^{88} \\
& + 11X^{90}Y^{88} + X^{91}Y^{88} + X^{80}Y^{89} + 4X^{81}Y^{89} + 26X^{82}Y^{89} + 54X^{83}Y^{89} \\
& + 82X^{84}Y^{89} + 6X^{85}Y^{89} - 47X^{86}Y^{89} - 111X^{87}Y^{89} - 123X^{88}Y^{89} \\
& - 41X^{89}Y^{89} - X^{90}Y^{89} + 12X^{91}Y^{89} + X^{92}Y^{89} - X^{81}Y^{90} + 10X^{82}Y^{90} \\
& + 5X^{83}Y^{90} + 58X^{84}Y^{90} + 67X^{85}Y^{90} + 84X^{86}Y^{90} + 3X^{87}Y^{90} - 96X^{88}Y^{90} \\
& - 93X^{89}Y^{90} - 62X^{90}Y^{90} - 3X^{91}Y^{90} + 6X^{92}Y^{90} - 2X^{82}Y^{91} + 4X^{83}Y^{91} \\
& + 9X^{84}Y^{91} + 58X^{85}Y^{91} + 66X^{86}Y^{91} + 99X^{87}Y^{91} + 40X^{88}Y^{91} - 59X^{89}Y^{91} \\
& - 73X^{90}Y^{91} - 66X^{91}Y^{91} - 8X^{92}Y^{91} + 4X^{93}Y^{91} + X^{94}Y^{91} + X^{83}Y^{92} \\
& - 6X^{84}Y^{92} + 10X^{85}Y^{92} + 31X^{86}Y^{92} + 65X^{87}Y^{92} + 114X^{88}Y^{92} + 69X^{89}Y^{92} \\
& + 22X^{90}Y^{92} - 89X^{91}Y^{92} - 51X^{92}Y^{92} - 12X^{93}Y^{92} + 2X^{94}Y^{92} + X^{84}Y^{93} \\
& - 5X^{85}Y^{93} + 3X^{86}Y^{93} + 20X^{87}Y^{93} + 66X^{88}Y^{93} + 109X^{89}Y^{93} + 55X^{90}Y^{93} \\
& + 70X^{91}Y^{93} - 60X^{92}Y^{93} - 40X^{93}Y^{93} - 19X^{94}Y^{93} + X^{95}Y^{93} - X^{85}Y^{94} \\
& - 2X^{86}Y^{94} - 7X^{87}Y^{94} + 4X^{88}Y^{94} + 43X^{89}Y^{94} + 85X^{90}Y^{94} + 118X^{91}Y^{94} \\
& + 59X^{92}Y^{94} - 9X^{93}Y^{94} - 43X^{94}Y^{94} - 18X^{95}Y^{94} + X^{96}Y^{94} - X^{86}Y^{95} \\
& - 3X^{87}Y^{95} - 15X^{88}Y^{95} + 3X^{89}Y^{95} + 34X^{90}Y^{95} + 51X^{91}Y^{95} + 131X^{92}Y^{95} \\
& + 39X^{93}Y^{95} + 31X^{94}Y^{95} - 29X^{95}Y^{95} - 17X^{96}Y^{95} - 2X^{97}Y^{95} - 4X^{88}Y^{96} \\
& - 17X^{89}Y^{96} - 10X^{90}Y^{96} - 3X^{91}Y^{96} + 45X^{92}Y^{96} + 94X^{93}Y^{96} + 95X^{94}Y^{96} \\
& + 29X^{95}Y^{96} - 18X^{96}Y^{96} - 17X^{97}Y^{96} - X^{98}Y^{96} - 6X^{89}Y^{97} - 10X^{90}Y^{97} \\
& - 17X^{91}Y^{97} - 28X^{92}Y^{97} + 63X^{93}Y^{97} + 31X^{94}Y^{97} + 99X^{95}Y^{97} + 26X^{96}Y^{97} \\
& + 5X^{97}Y^{97} - 13X^{98}Y^{97} - X^{99}Y^{97} - 5X^{90}Y^{98} - 11X^{91}Y^{98} - 22X^{92}Y^{98} \\
& - 33X^{93}Y^{98} - 4X^{94}Y^{98} + 45X^{95}Y^{98} + 86X^{96}Y^{98} + 48X^{97}Y^{98} + 3X^{98}Y^{98} \\
& - 11X^{99}Y^{98} - X^{90}Y^{99} - 2X^{91}Y^{99} - 10X^{92}Y^{99} - 34X^{93}Y^{99} - 10X^{94}Y^{99} \\
& - 48X^{95}Y^{99} + 47X^{96}Y^{99} + 41X^{97}Y^{99} + 56X^{98}Y^{99} + 5X^{99}Y^{99} - 3X^{100}Y^{99} \\
& - 2X^{101}Y^{99} - 10X^{93}Y^{100} - 27X^{94}Y^{100} - 32X^{95}Y^{100} - 43X^{96}Y^{100}
\end{aligned}$$

$$\begin{aligned}
& + 34X^{98}Y^{100} + 66X^{99}Y^{100} + 6X^{100}Y^{100} - 4X^{101}Y^{100} - 10X^{94}Y^{101} \\
& - 4X^{95}Y^{101} - 57X^{96}Y^{101} - 20X^{97}Y^{101} - 27X^{98}Y^{101} + 13X^{99}Y^{101} \\
& + 36X^{100}Y^{101} + 20X^{101}Y^{101} - X^{103}Y^{101} + X^{94}Y^{102} - 4X^{95}Y^{102} \\
& - 11X^{96}Y^{102} - 37X^{97}Y^{102} - 47X^{98}Y^{102} - 44X^{99}Y^{102} + 18X^{100}Y^{102} \\
& + 27X^{101}Y^{102} + 17X^{102}Y^{102} + X^{103}Y^{102} - 2X^{104}Y^{102} - 2X^{95}Y^{103} \\
& + 6X^{96}Y^{103} - 17X^{97}Y^{103} - 12X^{98}Y^{103} - 47X^{99}Y^{103} - 39X^{100}Y^{103} \\
& - 10X^{101}Y^{103} + 16X^{102}Y^{103} + 12X^{103}Y^{103} + 5X^{104}Y^{103} - X^{105}Y^{103} \\
& + X^{96}Y^{104} + 4X^{97}Y^{104} - 4X^{98}Y^{104} - 11X^{99}Y^{104} - 50X^{100}Y^{104} \\
& - 38X^{101}Y^{104} - 28X^{102}Y^{104} + 20X^{103}Y^{104} + 13X^{104}Y^{104} - X^{105}Y^{104} \\
& + 2X^{97}Y^{105} + 8X^{99}Y^{105} - 9X^{100}Y^{105} - 28X^{101}Y^{105} - 32X^{102}Y^{105} \\
& - 33X^{103}Y^{105} - 2X^{104}Y^{105} + 11X^{105}Y^{105} + 2X^{106}Y^{105} + 2X^{98}Y^{106} \\
& + 4X^{99}Y^{106} + 10X^{100}Y^{106} - 5X^{101}Y^{106} - 11X^{102}Y^{106} - 44X^{103}Y^{106} \\
& - 33X^{104}Y^{106} + 5X^{105}Y^{106} + 5X^{106}Y^{106} + 2X^{107}Y^{106} + 6X^{100}Y^{107} \\
& + 7X^{101}Y^{107} + 4X^{102}Y^{107} + X^{103}Y^{107} - 21X^{104}Y^{107} - 37X^{105}Y^{107} \\
& - 7X^{106}Y^{107} + 5X^{107}Y^{107} + X^{108}Y^{107} + X^{100}Y^{108} + 7X^{101}Y^{108} \\
& + 5X^{102}Y^{108} + 14X^{103}Y^{108} - 4X^{104}Y^{108} - 15X^{105}Y^{108} - 24X^{106}Y^{108} \\
& - 15X^{107}Y^{108} + 8X^{108}Y^{108} - X^{109}Y^{108} + X^{101}Y^{109} + 3X^{102}Y^{109} \\
& + 5X^{103}Y^{109} + 20X^{104}Y^{109} + 6X^{105}Y^{109} - 3X^{106}Y^{109} - 19X^{107}Y^{109} \\
& - 16X^{108}Y^{109} + 2X^{110}Y^{109} + X^{102}Y^{110} + 2X^{103}Y^{110} + 7X^{104}Y^{110} \\
& + 17X^{105}Y^{110} + 8X^{106}Y^{110} + 2X^{107}Y^{110} - 23X^{108}Y^{110} - 4X^{110}Y^{110} \\
& + X^{111}Y^{110} + 5X^{105}Y^{111} + 7X^{106}Y^{111} + 20X^{107}Y^{111} + 16X^{108}Y^{111} \\
& - 16X^{109}Y^{111} - 8X^{110}Y^{111} - X^{111}Y^{111} + 7X^{106}Y^{112} + 7X^{107}Y^{112} \\
& + 16X^{108}Y^{112} + 4X^{109}Y^{112} + 2X^{110}Y^{112} - 12X^{111}Y^{112} + 2X^{112}Y^{112} \\
& + X^{106}Y^{113} - 2X^{107}Y^{113} + 6X^{108}Y^{113} + 16X^{109}Y^{113} + 14X^{110}Y^{113} \\
& + 4X^{111}Y^{113} - 10X^{112}Y^{113} - 2X^{113}Y^{113} + X^{114}Y^{113} - X^{108}Y^{114} \\
& + 7X^{109}Y^{114} + 4X^{110}Y^{114} + 19X^{111}Y^{114} - 4X^{112}Y^{114} + X^{113}Y^{114} \\
& - 2X^{114}Y^{114} - 2X^{108}Y^{115} - X^{109}Y^{115} + 2X^{111}Y^{115} + 22X^{112}Y^{115} \\
& + X^{113}Y^{115} - 3X^{114}Y^{115} - 2X^{109}Y^{116} + X^{110}Y^{116} - 3X^{111}Y^{116} \\
& + 9X^{112}Y^{116} - 2X^{113}Y^{116} + 10X^{114}Y^{116} - 2X^{115}Y^{116} - X^{110}Y^{117} \\
& - X^{111}Y^{117} - 11X^{112}Y^{117} + 10X^{113}Y^{117} + 5X^{114}Y^{117} + 7X^{115}Y^{117} \\
& - X^{116}Y^{117} - X^{117}Y^{117} - X^{111}Y^{118} - 2X^{112}Y^{118} + X^{113}Y^{118} \\
& - 6X^{114}Y^{118} + 6X^{115}Y^{118} + 2X^{116}Y^{118} + X^{117}Y^{118} - 4X^{113}Y^{119} \\
& - X^{114}Y^{119} - 9X^{115}Y^{119} + 6X^{116}Y^{119} + 6X^{117}Y^{119} - X^{118}Y^{119} \\
& - X^{113}Y^{120} - 3X^{115}Y^{120} + X^{116}Y^{120} - 5X^{117}Y^{120} + 4X^{118}Y^{120}
\end{aligned}$$

$$\begin{aligned}
& + 2X^{115}Y^{121} - 7X^{116}Y^{121} - 2X^{117}Y^{121} - 2X^{118}Y^{121} + 5X^{119}Y^{121} \\
& - X^{120}Y^{121} - X^{116}Y^{122} - 2X^{118}Y^{122} - 3X^{119}Y^{122} + X^{120}Y^{122} \\
& - X^{117}Y^{123} - 5X^{119}Y^{123} + X^{121}Y^{123} + X^{119}Y^{124} - 2X^{120}Y^{124} \\
& - 2X^{121}Y^{124} + X^{119}Y^{125} - 2X^{121}Y^{125} - 2X^{122}Y^{125} + X^{123}Y^{125} \\
& + X^{121}Y^{126} - X^{123}Y^{126} + X^{122}Y^{127} - X^{123}Y^{127} - X^{124}Y^{127} + X^{123}Y^{128} \\
& + X^{124}Y^{129} + X^{127}Y^{132}.
\end{aligned}$$

$\zeta_{\mathfrak{g}_{247B}}^{\triangleleft}(s)$  is uniform.

### 3 Functional equation

The local zeta function satisfies the functional equation

$$\zeta_{\mathfrak{g}_{247B},p}^{\triangleleft}(s)\Big|_{p \rightarrow p^{-1}} = -p^{21-15s}\zeta_{\mathfrak{g}_{247B},p}^{\triangleleft}(s).$$

### 4 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{\mathfrak{g}_{247B}}^{\triangleleft}(s)$  is 3, with a simple pole at  $s = 3$ .

### 5 Ghost zeta function

The ghost zeta function is the product over all primes of

$$\begin{aligned}
& \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(3s-4)\zeta_p(4s-3)\zeta_p(5s-5)\zeta_p(5s-6)\zeta_p(6s-5) \\
& \times \zeta_p(6s-6)\zeta_p(7s-6)\zeta_p(7s-7)\zeta_p(8s-7)\zeta_p(8s-8)\zeta_p(9s-10)\zeta_p(9s-11) \\
& \times \zeta_p(10s-9)\zeta_p(10s-11)\zeta_p(11s-10)\zeta_p(11s-12)\zeta_p(12s-12)\zeta_p(13s-13) \\
& \times W_1(p, p^{-s})W_2(p, p^{-s})W_3(p, p^{-s})W_4(p, p^{-s})W_5(p, p^{-s})W_6(p, p^{-s}) \\
& \times W_7(p, p^{-s})
\end{aligned}$$



where

$$\begin{aligned}
W_1(X, Y) &= 1 - X^{14}Y^{12}, \\
W_2(X, Y) &= -1 + X^{11}Y^{10}, \\
W_3(X, Y) &= 1 - X^{12}Y^{11}, \\
W_4(X, Y) &= -1 - X^4Y^4 - X^5Y^5 + X^6Y^6 + X^7Y^7 - 2X^8Y^8 - X^9Y^9 \\
&\quad + X^{10}Y^{10} + 2X^{11}Y^{11} + X^{12}Y^{12} + X^{13}Y^{13} + X^{14}Y^{14} \\
&\quad + 2X^{15}Y^{15} + X^{16}Y^{16} + 2X^{17}Y^{17} + X^{18}Y^{18} - X^{19}Y^{19} \\
&\quad - X^{20}Y^{20} + X^{21}Y^{21} - 3X^{23}Y^{23} - 3X^{24}Y^{24} - 3X^{25}Y^{25} \\
&\quad - X^{26}Y^{26} - 2X^{27}Y^{27} - 2X^{28}Y^{28} - X^{29}Y^{29} + X^{30}Y^{30} \\
&\quad + X^{31}Y^{31} + X^{32}Y^{32} + 2X^{33}Y^{33} + 2X^{34}Y^{34} + X^{36}Y^{36} \\
&\quad + X^{37}Y^{37} + X^{38}Y^{38} + X^{40}Y^{40} - X^{43}Y^{43} - X^{44}Y^{44} \\
&\quad - X^{45}Y^{45} - X^{46}Y^{46} + X^{51}Y^{51}, \\
W_5(X, Y) &= 1 + X^{26}Y^{29}, \\
W_6(X, Y) &= 1 - X^3Y^4 - X^6Y^8 + X^9Y^{12}, \\
W_7(X, Y) &= 1 + X^4Y^7.
\end{aligned}$$

The ghost is unfriendly.

## 6 Natural boundary

$\zeta_{\mathfrak{g}_{247B}}^{\triangleleft}(s)$  has a natural boundary at  $\Re(s) = 7/6$ , and is of type III.