

# The zeta function of $\mathfrak{g}_{1357C}$ counting ideals

## 1 Presentation

$\mathfrak{g}_{1357C}$  has presentation

$$\left\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \left| \begin{array}{l} [x_1, x_2] = x_4, [x_1, x_4] = x_5, [x_1, x_5] = x_7, [x_2, x_3] = x_5, \\ [x_2, x_4] = x_7, [x_3, x_4] = -x_7, [x_3, x_6] = x_7 \end{array} \right. \right\rangle.$$

$\mathfrak{g}_{1357C}$  has nilpotency class 4.

## 2 The local zeta function

The local zeta function was first calculated by Luke Woodward. It is

$$\zeta_{\mathfrak{g}_{1357C}, p}^{\triangleleft}(s) = \zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(3s-4)\zeta_p(5s-5)\zeta_p(7s-4) \\ \times \zeta_p(9s-6)\zeta_p(11s-10)\zeta_p(16s-11)W(p, p^{-s})$$

where  $W(X, Y)$  is

$$1 - X^4Y^8 + X^5Y^8 - X^5Y^9 - X^9Y^{11} + X^9Y^{12} - X^{10}Y^{12} - X^{10}Y^{16} \\ + X^{10}Y^{17} - X^{11}Y^{17} + X^{14}Y^{19} - X^{15}Y^{19} + X^{15}Y^{20} + X^{15}Y^{25} + X^{19}Y^{27} \\ - X^{19}Y^{28} + X^{21}Y^{28} - X^{25}Y^{36}.$$

$\zeta_{\mathfrak{g}_{1357C}}^{\triangleleft}(s)$  is uniform.

## 3 Functional equation

The local zeta function satisfies no functional equation.

## 4 Abscissa of convergence and order of pole

The abscissa of convergence of  $\zeta_{\mathfrak{g}_{1357C}}^{\triangleleft}(s)$  is 4, with a simple pole at  $s = 4$ .

## 5 Ghost zeta function

The ghost zeta function is the product over all primes of

$$\zeta_p(s)\zeta_p(s-1)\zeta_p(s-2)\zeta_p(s-3)\zeta_p(3s-4)\zeta_p(5s-5)\zeta_p(7s-4)\zeta_p(9s-6) \\ \times \zeta_p(11s-10)\zeta_p(16s-11)W_1(p, p^{-s})W_2(p, p^{-s})W_3(p, p^{-s})W_4(p, p^{-s})$$

where

$$\begin{aligned}W_1(X, Y) &= 1 - X^{10}Y^{12}, \\W_2(X, Y) &= -1 - X^5Y^7, \\W_3(X, Y) &= -1 + X^6Y^9, \\W_4(X, Y) &= 1 - X^4Y^8.\end{aligned}$$

The ghost is friendly.

## 6 Natural boundary

$\zeta_{\mathfrak{g}_{1357C}}^{\triangleleft}(s)$  has a natural boundary at  $\Re(s) = 5/6$ , and is of type III.

## 7 Notes

This ideal zeta function is identical to that of  $\mathfrak{g}_{1357B}$ , though the Lie rings themselves are non-isomorphic.